One-loop inert and pseudo-inert minima

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We analyse the differences between inert and pseudo-inert vacua in the 2HDM, both at tree-level and at one-loop. The validity of tree-level formulae for the relative depth of the potential at both minima is studied. The one-loop analysis shows both minima can coexist in regions of parameter space forbidden at tree-level.

I. INTRODUCTION

The two Higgs doublet model (2HDM) [1] is one of the simplest extensions of the Standard Model (SM) of particle physics, in which an extra scalar doublet is added to the theory. This addition originates a richer scalar spectrum than the SM's, and in the versions of the model wherein CP symmetry is conserved, this includes two CP-even scalars (the lightest h and the heaviest H), a pseudoscalar, A, and a charged scalar, $H^{\pm}$. For a recent 2HDM review, see [2]. The recent discovery of the Higgs boson [3, 4] constrained the 2HDM parameter space, and it has been verified that the model survives comparison with data. In fact, the 2HDM does a very good job describing the LHC results [5–9]. The 2HDM has an interesting phenomenology, including possible spontaneous CP violation, tree-level scalar-mediated flavour changing neutral currents and, in certain versions of the model, dark matter candidates.

Those versions of the 2HDM correspond to a theory wherein one has imposed a discrete Z_2 symmetry on the potential, the so-called *Inert Doublet Model* (IDM) [10–12]. Under such a symmetry, one of the doublets (Φ_1, for instance) is left unchanged, but the other (Φ_2) is transformed, such that $\Phi_2 \to -\Phi_2$. Then, in the IDM, the electroweak symmetry is broken by a vacuum which preserves the discrete Z_2, and as such there is a new quantum number which must be preserved in all interactions. By choosing the Z_2 parity of the particles of the model, it is simple to ensure that several of the scalars do not couple to fermions at tree-level (nor indeed possess any triple couplings to gauge bosons). This leads to the lightest neutral scalar which is odd under Z_2 being stable, and as such a prime candidate for dark matter. For works on the IDM, see for instance [13–31].

The Z_2-symmetric 2HDM scalar potential, furthermore, has an interesting vacuum structure. Several types of extrema are possible already at tree-level, and under specific circumstances, minima of different depths, leading to different types of physics, may coexist in the model [23, 34–36]. In the current work, we will review the tree-level analysis pertaining to the coexistence of such minima, and study the impact that one-loop contributions to the potential might have upon those conclusions. In order to do so, we will use the effective potential formalism to undertake the computation of the one-loop potential in a simplified theory without fermions or gauge bosons.

II. THE VACUUM STRUCTURE OF THE INERT MODEL

The most general 2HDM potential has 14 real parameters, although these may be reduced to 11 using the reparametrization invariance of the model. In order to avoid tree-level flavour changing neutral currents — which are very strongly constrained by experimental measurements — Glashow, Weinberg and Paschos [32, 33] proposed the imposition of a discrete Z_2 symmetry on the lagrangian, so that $\Phi_1 \to \Phi_1$ and $\Phi_2 \to -\Phi_2$. The resulting scalar potential has but seven independent real parameters and is written as

$$V = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right]. \quad (1)$$

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So that the potential is bounded from below — thus guaranteed to possess a stable minimum — the quartic couplings must obey [10]

$$\begin{aligned} &\lambda_1 > 0 \ , \ \lambda_2 > 0 \ , \\ \lambda_3 > -\sqrt{\lambda_1\lambda_2} \ , \ &\lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2} \ . \end{aligned} \quad (2)$$

The Z_2 symmetry must be applied to the whole 2HDM lagrangian, otherwise the model would not be renormalisable. Typically one chooses the Z_2 "charges" of the fermions in such a way that, for instance, only Φ_2 couples to all fermions (model type I); or such that Φ_2 couples to up-type quarks, Φ_1 to the remaining fermions [40]. In the IDM, traditionally, the doublet which is made to couple to all fermions is Φ_1, and Φ_2 has no coupling to fermions at all.

Let us now consider the possibility of spontaneous symmetry breaking in the potential of Eq. 1. The doublets may develop neutral vacuum expectation values (vevs), such that $\langle\Phi_1\rangle = (0\,,\, v_1)^T/\sqrt{2}$ and $\langle\Phi_2\rangle = (0\,,\, v_2)^T/\sqrt{2}$. Depending on the values of the parameters of the potential, there are three possible neutral extrema [10]:

- Both v_1 and v_2 are non-zero and the vacuum breaks the Z_2 symmetry. This is the "normal" 2HDM vacuum, interesting in its own right, but not what we wish to consider here.
- The vev v_1 is non-zero, but $v_2 = 0$. This is the *inert vacuum*: the Z_2 symmetry is preserved, the fermions acquire a mass through v_1 and the Higgs-like scalar h is the real neutral component of Φ_1. The remaining components of Φ_1 correspond to the Goldstone bosons G and $G^\pm$, and the doublet Φ_2's components originate the rest of the scalars — H, A and $H^\pm$, none of which couple to fermions. The lightest of these is the dark matter candidate [41].
- The vev v_2 is non-zero, but $v_1 = 0$. This is the *pseudo-inert vacuum*, or *inert-like vacuum*: a Z_2 symmetry is also preserved, but since the fermions only couple to Φ_1 they are massless in this vacuum. Thus this vacuum is not an acceptable description of reality, and should be avoided as unphysical.

Trivial calculations allow us to establish that, for the inert vacuum, one has

$$v_1^2 \;=\; -\,\frac{2\,m_{11}^2}{\lambda_1}, \quad \text{provided} \quad m_{11}^2 < 0. \quad (3)$$

Likewise, for the pseudo-inert vacuum, one must have

$$v_2^2 \;=\; -\,\frac{2\,m_{22}^2}{\lambda_2}, \quad \text{provided} \quad m_{22}^2 < 0. \quad (4)$$

It has been shown [34–36] that minima which break different symmetries cannot coexist. As such, if the potential has parameters such that a minimum with non-zero $(v_1\,,\, v_2)$ exists, then no inert or pseudo-inert minimum exists. However, inert and pseudo-inert vacua can and do coexist in the model, provided the following conditions are met, at tree-level:

Inert and pseudo-inert minima can coexist in the potential if $m_{11}^2 < 0$ *and* $m_{22}^2 < 0$. (5)

It is simple to show that there exists an analytical relation between the values of the potential at the inert minimum (V_I) and the pseudo-inert one (V_{PI}). Let $v_1 = v$ be the vev value at the inert minimum, and $v_2 = v'$ the vev at the coexisting pseudo-inert vacuum. Then, we have

$$V_I \,-\, V_{PI} \;=\; \frac{1}{2}\left(\frac{m_{22}^4}{\lambda_2} - \frac{m_{11}^4}{\lambda_1}\right) \quad (6)$$

$$\;=\; \frac{1}{4}\left[\left(\frac{m_{H^\pm}^2}{v'^2}\right)_{PI} - \left(\frac{m_{H^\pm}^2}{v^2}\right)_{I}\right] v^2\, v'^2 \quad (7)$$

where in the second formula we use the value of the squared charged masses at each minimum: in the inert one, we have

$$\left(m_{H^\pm}^2\right)_I = m_{22}^2 + \frac{1}{2}\,\lambda_3\, v^2 \ . \quad (8)$$

An analogous expression, with the exchanges $m_{22} \leftrightarrow m_{11}$ and $v \leftrightarrow v'$, is valid for the charged mass at the pseudo-inert vacuum.

These expressions relating the relative depths of the potential at each of the coexisting minima are obtained at tree-level. They show that none of the two minima is preferred to the other - depending on the model's parameters, either minima can be the global one of the theory. In the rest of this work, we wish to analyse how they change when the one-loop contributions to the potential are taken into account. In particular, we wish to investigate whether an "inversion" of the inert and pseudo-inert minima depths can occur once the one-loop contributions are considered.

III. THE ONE-LOOP 2HDM POTENTIAL

At one-loop, the effective potential is given by (in the $\overline{MS}$ scheme, in the Landau gauge)

$$V \,=\, V_0 \,+\, V_1\,, \tag{9}$$

with V_0 given by Eq. 1 and the one-loop contribution equal to

$$V_1 = \frac{1}{64\pi^2}\sum_\alpha n_\alpha\, m^4_\alpha(\varphi_i)\left[\log\left(\frac{m^2_\alpha(\varphi_i)}{\mu^2}\right) - \frac{3}{2}\right] \tag{10}$$

where μ is the renormalization scale chosen and the $m_\alpha(\varphi_i)$ are the field-dependent mass eigenvalues of all particles present in the theory. In the following analysis we have fixed the value of μ to be 200 GeV, a mass scale close enough to the mass scales of the scalars we will be considering. This should be enough for our purposes — let us recall that the effective potential of Eq. 9 is renormalization scale independent up to two-loop effects and a good perturbative approximation provided μ is of the order of the mass scales involved (thus rendering the logarithms in Eq. 10 "small").

In all that follows, we will consider a 2HDM without fermions or gauge bosons. In other words, a model with two hypercharge 1 doublets, with a global $SU(2)_W \times U(1)_Y$ symmetry. This toy model, as we will show, will allow us to ascertain the main features of the one-loop contributions which interest us. In future works a realistic model will be considered. The sum over α in Eq. 10 runs therefore from 1 to 8 (all the scalar eigenstates, though some of them are degenerate, such as the two charged goldstones and charged scalars). The φ_i are the eight real components of the doublets. The factor n_α counts the number of degrees of freedom corresponding to each particle, and is in general given, for a particle of spin s_α, by

$$n_\alpha \,=\, (-1)^{2s_\alpha}\, Q_\alpha C_\alpha(2s_\alpha + 1), \tag{11}$$

where Q_α is 1 for uncharged particles and 2 for charged ones; C_α counts the number of colour degrees of freedom (for particles without colour it equals 1, for particles with colour, 3). Then, the first derivatives of the one-loop potential are given by (dropping the explicit field dependence in the masses for simplicity of notation)

$$\frac{\partial V}{\partial\varphi_i} = \frac{\partial V_0}{\partial\varphi_i} + \frac{1}{32\pi^2}\sum_\alpha m^2_\alpha\,\frac{\partial m^2_\alpha}{\partial\varphi_i}\left[\log\left(\frac{m^2_\alpha}{\mu^2}\right) - 1\right]. \tag{12}$$

Equation (12) can be considerably simplified for the computation of the inert (and pseudo-inert) vacuum. In the inert case, we have $\langle r_1\rangle = v_1/\sqrt{2}$, and all remaining $\varphi_i = 0$. An explicit calculation has shown that all derivatives of the one-loop potential with respect to the fields φ_i, except the one with respect to the real neutral component of Φ_1, are trivially equal to zero for the inert minimum. Due to the conventions we have chosen, performing this derivative is equivalent to differentiating with respect to v_1. Thus we obtain [42]

$$\begin{aligned}\frac{1}{v_1}\frac{\partial V}{\partial v_1} \;=\;& m^2_{11} + \frac{1}{2}\lambda_1\, v_1^2\, + \\ & \frac{1}{32\pi^2}\left\{\lambda_1\, m^2_{G_0}\left[\log\left(\frac{m^2_{G_0}}{\mu^2}\right) - 1\right] + 3\,\lambda_1\, m^2_{h_0}\left[\log\left(\frac{m^2_{h_0}}{\mu^2}\right) - 1\right] + \right.\\ & \lambda_{345}\, m^2_{H_0}\left[\log\left(\frac{m^2_{H_0}}{\mu^2}\right) - 1\right] + \bar{\lambda}_{345}\, m^2_{A_0}\left[\log\left(\frac{m^2_{A_0}}{\mu^2}\right) - 1\right] + \\ & \left. 2\,\lambda_3\, m^2_{H_0^\pm}\left[\log\left(\frac{m^2_{H_0^\pm}}{\mu^2}\right) - 1\right] + 2\,\lambda_1\, m^2_{G_0^\pm}\left[\log\left(\frac{m^2_{G_0^\pm}}{\mu^2}\right) - 1\right]\right\} \;=\; 0,\end{aligned} \tag{13}$$

where the tree-level scalar masses at the inert minimum are given by

$$m^2_{G_0} = m^2_{11} + \frac{1}{2}\,\lambda_1\, v_1^2, \qquad m^2_{G_0^\pm} = m^2_{11} + \frac{1}{2}\,\lambda_1\, v_1^2, \tag{14}$$

$$m^2_{h_0} = m^2_{11} + \frac{3}{2}\,\lambda_1\, v_1^2, \qquad m^2_{H_0} = m^2_{22} + \frac{1}{2}\,\lambda_{345}\, v_1^2, \tag{15}$$

$$m^2_{A_0} = m^2_{22} + \frac{1}{2}\,\bar{\lambda}_{345}\, v_1^2, \qquad m^2_{H_0^\pm} = m^2_{22} + \frac{1}{2}\,\lambda_3\, v_1^2, \tag{16}$$

At the tree-level inert minimum, these tree-level Goldstone masses would be identically zero. At the one-loop minimum, however, that is no longer so — one must compute the full one-loop expressions for m^2_G and $m^2_{G^\pm}$ and verify that they are zero using the full one-loop minimization conditions. We have performed that check and are thus confident of our one-loop minimization procedure.

In order to ensure we are at a one-loop minimum, all one-loop scalar masses had to be computed. We worked under the effective potential approximation, assuming that the squared scalar masses are given by the second derivatives of the one-loop potential. This has been proven [37–39] to be a very good approximation to the true one-loop masses. The calculation is made more difficult by the need to maintain the field dependency in the eigenvalues $m_\alpha(\varphi_i)$ when performing the derivatives.

IV. ONE-LOOP INERT AND PSEUDO-INERT MINIMA

The one-loop contributions to inert vacua have been studied in great detail (using a different renormalization approach) in [26]. However, in that work the main topic of analysis was $T \neq 0$ contributions to the effective potential and its phenomenology. Here we are interested in the possibility of minima inversion going from the tree-level to the one-loop potential. Our procedure consisted in scanning the 2HDM parameter space, computing the one-loop effective potential and its (first and second) derivatives, such that:

- All tree-level bounded from below conditions were obeyed. This should, to first approximation, ensure the one-loop potential is also limited from below.
- Simultaneous inert and pseudo-inert vacua coexist for the choice of parameters made. The inert vacuum is simply defined as the one with $v_1 = 246$ GeV and $v_2 = 0$, the pseudo-inert vacuum is such that $v_1 = 0$, $v_2 \neq 0$.
- We then computed all squared scalar masses at both vacua and demanded they are all positive (minus the Goldstone boson masses at both minima, which we verified are equal to zero). We are then assured that we have coexisting minima.

With a scan of over 4000 points in 2HDM parameter space with coexisting minima, the comparison of the one-loop potential inert and pseudo-inert minima depths is shown in figure 1. In this plot, we show the difference in value of the inert and pseudo-inert one-loop minima ($V_I - V_{PI}$) against the tree-level expected difference in depths from Eq. 6, *i.e.*

$$\Delta V_0^{(1)} = \frac{1}{2}\left(\frac{m^4_{22}}{\lambda_2} - \frac{m^4_{11}}{\lambda_1}\right). \tag{17}$$

The conclusions to draw from figure 1 are several:

- The tree-level formula from Eq. 6 is a very good approximation to the one-loop potential depth difference.
- It is not however *perfect*, since there are clearly deviations from it in the one-loop results.
- Furthermore, one finds points for which the inert and pseudo-inert minima are *inverted* going from tree-level to one-loop: if at tree-level the inert minimum was expected to be the deepest, that is no longer so at one-loop.
- Confirming that perturbation theory still holds, such inversions are *rare* (they have only occurred for about 3% of all simulated points) and only occur when both minima are close to degenerate.

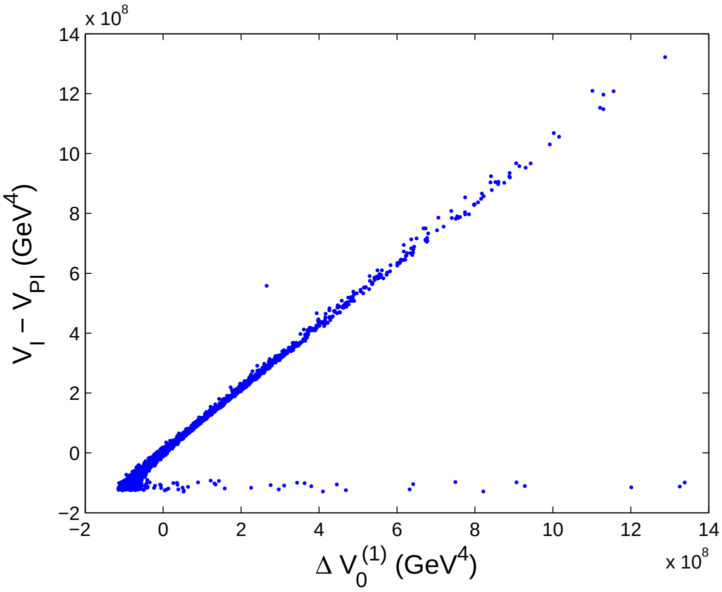

FIG. 1: One-loop computed difference in inert and pseudo-inert minima depths ($V_I - V_{PI}$) *versus* the tree-level expected depth difference given by Eq. 6.

An interesting observation, though, is obtained if one performs the comparison between $V_I - V_{PI}$ and the tree-level depth difference formula provided by Eq. 7, *i.e.*, we are now comparing with

$$\Delta V_0^{(2)} = \frac{1}{4}\left[\left(\frac{m^2_{H^\pm}}{v'^2}\right)_{PI} - \left(\frac{m^2_{H^\pm}}{v^2}\right)_{I}\right] v^2\, v'^2. \tag{18}$$

We use this formula with the values of the one-loop charged masses. As one sees in figure 2, the one-loop

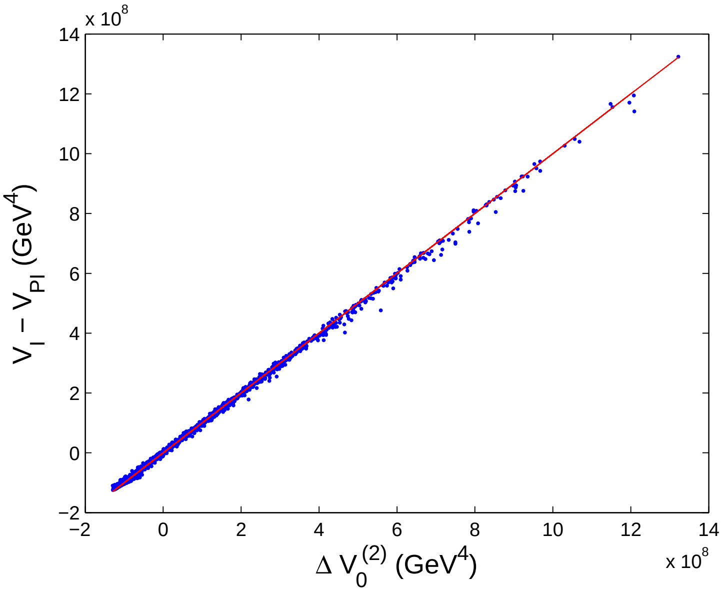

FIG. 2: One-loop computed difference in inert and pseudo-inert minima depths ($V_I - V_{PI}$) *versus* the tree-level expected depth difference now given by Eq. 7.

difference in potential depths is almost perfectly reproduced by Eq. 18 — the red line in the plot would correspond to the perfect equality $V_I - V_{PI} = \Delta V_0^{(2)}$, and we see there are very little deviations from it. And

in fact, the inversions in depth between both minima now only occur for 0.5% of the points simulated (once again, and reassuringly, only for nearly-degenerate potentials).

Equally interesting conclusions are drawn from figure 3, where we plot the mass of the lightest inert scalar

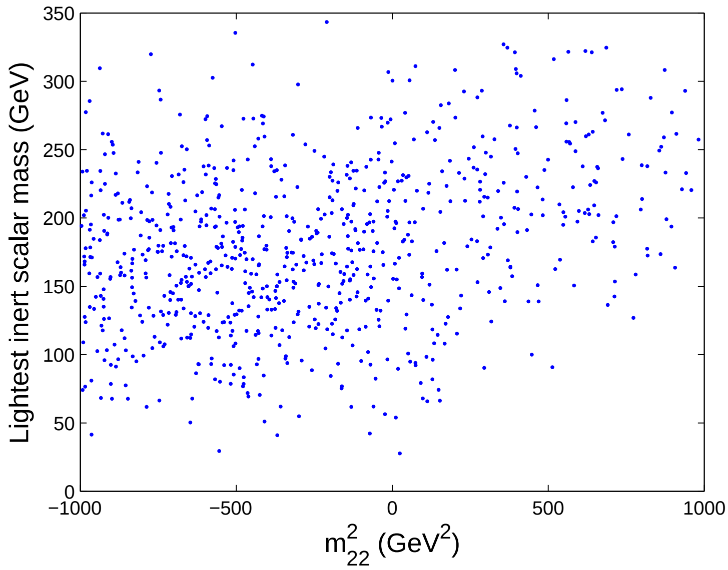

FIG. 3: Lightest inert scalar mass at the inert minimum *versus* the m_{22}^2 quadratic parameter.

at the inert vacuum (meaning, the dark matter candidate) against the quadratic parameter m_{22}^2, for all points scanned with coexisting inert and pseudo-inert minima. Though the range of masses for the dark matter candidate is interesting in its own merit, it's the horizontal axis that provides us with a very interesting fact: *at one-loop, we can have coexistence of inert and pseudo-inert minima even if* $m_{22}^2 > 0$. This, according to the statement presented in 5, was not possible at tree-level!

V. CONCLUSIONS

The one-loop analysis of inert and pseudo-inert coexisting minima has shown that an inversion of the relative depths of these minima can be caused by radiative corrections. The results obtained indicate that this possibility can occur when the tree-level minima are nearly degenerate, so that this result does not put into question the validity of the perturbative approximation. We have further shown that the region of 2HDM parameter space where one can expect coexistence of inert and pseudo-inert minima is *extended* at the one-loop level, compared to tree-level expectations.

The formulae deduced at tree-level for the depth difference at both minima do not remain valid at the one-loop level. However, expressing the difference in potential depths in terms of physical charged masses almost perfectly reproduces the one-loop results. This almost suggests that the tree-level formula of Eq. 7 might indeed end up valid at the one loop level — remember that we computed all one-loop masses within the effective potential approximation, so it is within the realm of possibilities that a completely accurate mass calculation may lead to full agreement between Eq. 7 and the one-loop results. That would suggest that the tree-level deduction of that equation had somehow "stumbled" upon an exact formula.

Finally, all of these conclusions were drawn in the context of a toy model devoid of gauge bosons and fermions. But the conclusions are interesting by themselves, since they show that within the scalar sector alone one may already expect deviations from the tree-level formulae deduced for the potential depths' difference. Certainly it is to be expected that the inclusion of fermions will lead to great differences in tree-level and one-loop comparisons between inert and pseudo-inert minima — the fact that in the pseudo-inert minima all fermions are massless, as opposed to what happens for the inert minimum, leads one to expect further inversions of the minima, for instance. But the fact remains that the analysis herein detailed already shows that such inversions are not caused by the fermionic or gauge sector of the theory, they're rather already present within the scalar sector at the one-loop level.

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[40] In other possibilities - models type X and Y - one varies the coupling of the doublets to the leptons.
[41] Usually taken to be neutral, though choices of parameters are possible such that the charged scalar is the lightest state.
[42] Notice the factor of "2" affecting both charged scalar contributions in Eq. 13, counting the number of states effectively present.